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Enhancing preschool teachers' self-efficacy and ability to notice mathematics through a professional development program

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ABSTRACT

Numerous studies have underscored the significance of teachers' professional development (PD). In this investigation, we explore the effects of a PD program on preschool teachers' self-efficacy and their ability to notice mathematics in preschool activities. The PD program was based on the fundamental components of effective professional growth. The study enlisted eighty-seven preschool teachers from twenty-one different preschools, with forty-four teachers participating in the PD sessions and forty-three serving as the control group. Overall, our PD program demonstrated a notable influence on preschool teachers' ability to notice the use of intellectual artefacts delineating mathematics activities. Testimonials from participating teachers indicated a heightened understanding of mathematics in preschool settings, resulting in increased self-efficacy and heightened attention to mathematical concepts in everyday activities.

Keywords: professional development, preschool, mathematics, noticing, self-efficacy, instruction

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INTRODUCTION

Background

Professional development (PD) empowers educators to enhance their skills and knowledge, serving as a pathway to improving teaching abilities, gaining deeper insights into students' needs, and embracing challenges within and beyond the classroom environment. Carpenter et al. (1989) emphasized the importance of teachers' PD three decades ago and demonstrated its influence on participants learning. Subsequent research, as discussed in a review article by Darling-Hammond et al. (2017), has further elaborated on this relationship. A PD program focused on mathematics teaching and learning aims to support teachers in delivering appropriate mathematics teaching (cf. Polly et al., 2018). To do that the PD program should influence teachers' understanding and their beliefs about teaching and learning (Ball et al., 2001; Berlin et al., 2021; Hill et al., 2005; Remillard, 2005; Richardson, 1996; Shulman, 1986; Thompson, 1992). Though mathematics teachers participate in several activities when they teach, they need multiple competencies to manage these activities. Tatto et al. (2008) give examples related to mathematical curricular knowledge, knowledge of planning for mathematics teaching and learning and enacting mathematics for learning. When teachers enact mathematics for learning they (for instance) need to analyze or evaluate students' mathematical solutions or arguments, analyzing the content of students' questions, diagnosing typical student responses, including misconceptions, explaining, or representing mathematical concepts or procedures (Tatto et al., 2008). To conduct these activities, teachers need knowledge that makes it possible to discover and see distinctions in mathematics communicated in different contexts. This knowledge is not the least important for preschool teachers, who often draw the children's attention to mathematics in the contexts in which the children interact (Björklund & Palmér, 2024).

Preschool teachers have a key role in society in general and in the school system specifically, thou several researchers argue for the importance of high-quality preschool activities and early years for later achievements in mathematics (cf. Melhuish et al., 2008). Countries aiming to raise the level of knowledge among their populations should invest in developing high-quality preschool education. High-quality preschool education needs teachers who understand teaching and learning in the specific context where they act and are confident in their role as mathematics teachers. In the present study, we will investigate the impact of a PD program focusing on preschool teachers' self-efficacy and ability to notice mathematics in preschool activities.

Theoretical Background

In the theoretical background supporting early mathematics development in preschool, preschool teachers' self-efficacy and PD are elaborated.

Supporting early mathematics development

There has been an increased focus on children's mathematical learning and development during the preschool years (cf. Palmèr & Björklund, 2023). One reason for this heightened interest may stem from the advantages that preschool children with foundational mathematical knowledge gain in subsequent learning, not only in mathematics but also in science, literacy, and technology (Duncan et al., 2007).

A fundamental aspect of early mathematics learning lies in the informal mathematical exposure children encounter within preschool settings (Björklund & Palmér, 2024). Some children do not naturally pay attention to numbers and counting, and there exists a reciprocal relationship between spontaneous attention to numbers and later counting ability (Hannula & Lehtinen, 2005). Hannula and Lehtinen (2005) as well as Björklund and Palmér (2024) argue for the importance of supporting children's early spontaneous attention to numbers and counting. To achieve this, preschool teachers require intellectual artefacts for noticing (Säljö, 2000; Santagata et al., 2021), reflecting, and making decisions during activities aimed at supporting children's mathematics learning. One important intellectual artefact is the language (Mercer, 2004). Language is an important artefact that helps us collect and communicate experiences with each other (Säljö, 2000), for instance, what mathematical activities occur in a preschool activity. To be able to notice children's mathematics learning preschool teacher needs a language, and concepts that help them to pay attention to nuances in the activities the children engage in (Kaiser & König, 2019; Santagata et al., 2021). Santagata et al. (2021) argue that studies adopting a socio-cultural perspective have broadened the examination of noticing by emphasizing the role of artefacts in framing and informing teachers. In socio-cultural theory, artefacts–both physical and intellectual/language–mediate reality for individuals within different communities of practice (Säljö, 2000), thereby assisting teachers in navigating the complexities of their professional environments. Previous research aimed at supporting the development of teacher noticing has focused on various aspects such as teachers' awareness of student thinking, instructional practices, classroom discourse, and mathematical content (Santagata et al., 2021). In our study, we concentrate on preschool teachers' development of their ability to notice mathematics during preschool activities. This is important because Blömeke et al. (2022) have demonstrated a direct correlation between mathematics teachers' ability to perceive, interpret, and make decisions and students' progression in mathematics.

Teacher efficacy beliefs

Several studies have demonstrated that teaching practices are contingent upon teachers' competence, beliefs, and attitudes (cf. Berlin et al., 2021). A person's beliefs about their ability to perform tasks, such as teaching, and achieve specific goals are conceptualized by Bandura (1997) as self-efficacy. A substantial body of research has involved teachers in assessing their ability to teach (Tschannen-Moran & Hoy, 2001, 2007; Samuelsson et al., 2022). This research presupposes that teachers' self-perceptions and their confidence in their teaching capabilities represent a collective evaluation of their capacity to achieve positive outcomes concerning students' engagement and learning.

Hoy and Spero (2005) argued that teachers' efficacy beliefs encompass forward-looking considerations tied to teachers' assessments of their competence rather than their actual effectiveness. Bandura (1997) emphasized four factors supporting the development of self-efficacy:

(a) enactive mastery experience,

(b) vicarious experience,

(c) verbal persuasion, and

(d) physiological and affective state.

The most significant aspect of obtaining information about teachers' confidence in their teaching effectiveness is their enactive mastery experience. According to Bandura (1977), positive experiences, such as effectively maintaining classroom order or providing clear assessments from a student's viewpoint, enhance one's sense of competence. Conversely, recurrent failures, such as chaotic lessons or difficulties in engaging with students, can diminish performance (Hoy & Spero, 2005). The foundation for alternative experiences is laid through observing how other teachers manage specific teaching tasks. These vicarious experiences are crucial because individuals typically do not rely exclusively on their successful efforts to evaluate teaching effectiveness (Bandura, 1977). Such experiences are especially valued in shaping future evaluations of teaching effectiveness (Labone, 2004). Verbal persuasion becomes highly effective when the goal is to influence behavior. Feedback provided by supervisors, fellow teachers, school leaders, and pre-service teachers carries particular importance in this regard. Bandura (1997) highlights that the impact of social persuasion relies on the expertise and credibility of the individual whose opinion is seeking to be influential. One's physiological and emotional state, including feelings of relief and positive emotions, plays a fundamental role in fostering self-efficacy and the anticipation of future success. Conversely, in challenging and stressful situations, the circumstances can offer valuable insights into the personal strengths and limitations of one's abilities (Bandura, 1997).

Development of a professional development program for the study

The PD program in the present study aimed to enhance preschool teachers' ability to notice mathematics in preschool activities. Darling-Hammond et al. (2017) found seven activities that occurred in effective PD programs:

(1) content focused,

(2) incorporates active learning,

(3) supports collaboration,

(4) uses models of effective practice,

(5) provides coaching and expert support,

(6) offers feedback and reflection, and

(7) is of sustained duration.

Drawing on the central components outlined in Darling-Hammond et al.'s (2017) review of effective PD, our PD program was designed with the following key components:

1. **Content-focused:** Our program centered on specific teaching strategies related to foundational concepts in early number sense and geometry, aligning with the curriculum. This focused approach supported teachers' learning and improvement in their teaching practices.
2. **Active learning:** Active learning was facilitated through the design and testing of teaching strategies by teachers themselves, fostering a highly contextualized professional learning experience. Teachers engaged in discussions, observed

preschool activities, and described the mathematics present, creating a learner-centered community of practice.

3. **Facilitate collaboration:** Our PD program provided opportunities for teachers to collaborate, share ideas, and develop a common knowledge base. Collaboration occurred during discussions of concepts and descriptions of mathematics observed in preschool activities, ensuring that new knowledge was embedded in the context of its application.
4. **Utilize models of effective teaching:** Our program emphasized the importance of utilizing research-based models of effective teaching, as well as embracing experiential models based on proven experience. Noticing was highlighted as crucial for supporting children's initial attention to numbers and counting, with intellectual artefacts utilized to perceive and interpret mathematics in preschool activities.
5. **Provide expert support:** Expert support on content and evidence-based teaching was provided to address individual teachers' needs. Feedback and reflection were integral parts of the PD program, helping teachers thoughtfully learn and improve their practice.
6. **Duration:** The PD program was structured to provide teachers with sufficient time to learn, practice, implement, and reflect on new strategies. An iterative process was employed to ensure that teachers had many opportunities for learning and development.

By incorporating these components, the PD program aimed to effectively support preschool teachers in enhancing their ability to notice mathematics in preschool activities and improve their teaching practices accordingly. Another important foundation for how the program was implemented was Bandura's (1997) four factors supporting the development of self-efficacy:

(a) enactive mastery experience,

(b) vicarious experience,

(c) verbal persuasion, and

(d) physiological and affective state.

All these processes are engaged in the program (presented in the method section).

Aim of the Study

This study aims to analyze the impact of a PD program on preschool teachers' ability to notice mathematics and their self-efficacy in supporting mathematics learning in preschool. The study was grounded in the following research questions:

1. Are there differences before and after the PD program for preschool teachers' ability to notice mathematics in preschool?
2. To what extent does the PD affect the development of preschool teachers' self-efficacy in supporting mathematics learning in preschool?
3. Is there a relationship between preschool teachers' ability to notice mathematics in preschool activities and their self-efficacy in supporting mathematics learning?
4. How do preschool teachers' post-programs reflect on their learning, enactment in practice (EP), and their children's learning experiences?

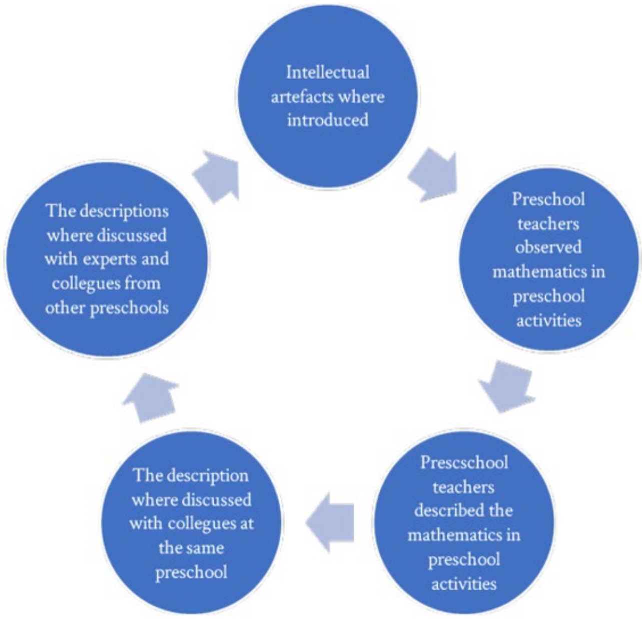

Figure 1. Description of each iteration process (Source: Author)

METHOD

Participants

There were eighty-seven preschool teachers from twenty-one preschools participating in the study. Forty-four preschool teachers participated in the PD program, and forty-three preschool teachers were assigned to the control group. All preschool teachers were educated at the university level to instruct preschool children.

Process of the Professional Development Program

The PD program was conducted over eight weeks, comprising five shared meetings between researchers and preschool teachers. Every meeting consisted of a lecture and discussions between preschool teachers and researchers, focusing on intellectual artefacts, concepts that could assist preschool teachers in noticing variations in mathematics during preschool activities. The concepts we discussed also appear in the Swedish curriculum for preschool. We presented concepts related to

(a) mathematical proficiency,

(b) basic arithmetic, and

(c) basic geometry.

Between our meetings the preschool teachers were instructed to describe the mathematics observed in preschool activities on a specific day and to send their reports to the researchers. This process was repeated for four iterations following the procedure outlined in **Figure 1**.

During the initial meeting, preschool teachers attended a lecture where they were presented with arguments emphasizing the importance of mastering intellectual artefacts to support mathematics learning in preschool children. Following the lecture, preschool teachers from each preschool engaged in discussions about the mathematics they encountered in their respective preschool settings.

The second lecture consisted of concepts related to mathematical proficiency. Kilpatrick et al. (2001) argue that no single term captures all aspects of mathematical knowledge. Kilpatrick et al. (2001) propose that mathematical proficiency consists of five strands:

- *Conceptual understanding–Comprehension of mathematical concepts, operations, and relations*
- *Procedural fluency–Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately*
- *Strategic competence–Ability to formulate, represent, and solve mathematical problems*
- *Adaptive reasoning–Capacity for logical thought, reflection, explanation, and justification*
- *Productive disposition–Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy* (p. 116).

The third lecture consisted of concepts related to basic arithmetic that occur in the national curriculum such as Gelman and Gallistel's (1978) five counting principles: the one-one principle, the stable-order principle, the cardinal principle, the abstraction principle, and the order-irrelevance principle.

- **The one-one principle:** Each item in a set to be counted gets only one number name.
- **The stable-order principle:** When counting, number words are always assigned in the same order.
- **The cardinal principle:** The number of objects in a set equals the final number counted.
- **The abstraction principle:** Anything can be counted using the abstract ideas in the preceding principles.
- **The order-irrelevance principle:** It doesn't matter where you start, provided that all items are counted.

The fourth lecture consisted of concepts related to geometry that occur in the National curriculum. In geometry, we discussed basic concepts that aid in describing object positions in the room (e.g., over, under, and behind), measurement, and shapes (e.g., circle and square) (cf. Clements, 2004). After all lectures the preschool teachers and the researchers discussed how these could occur in different preschool activities. On the fifth occasion, preschool teachers also composed a self-report regarding how their ability to notice mathematics influenced their practice enactment.

Data Collection Measures

To measure preschool teachers' ability to notice mathematics in preschool activities and their self-efficacy in supporting mathematics learning, three different methods of data collection were used.

The ability to notice mathematics was assessed through preschool teachers' descriptions of mathematics within preschool activities. In each iteration, they were asked to describe the mathematics that became apparent in preschool activities on a specific day. These written reports provided us with the opportunity to analyze the intellectual artefacts used by preschool teachers and the variety of intellectual artefacts present in the reports.

Preschool teachers' self-efficacy concerning supporting learning in mathematics was measured by nine items inspired by a translated version of the teachers' sense of efficacy scale developed by Tschannen-Moran and Hoy (2001), which also was used by Charalambous et al. (2007) and Samuelsson et al. (2022). Thus, self-efficacy was measured by 9 statements on a seven-point Likert scale where the preschool teachers were asked to what extent they agreed or disagreed with each statement (1 = disagreed; 7 = agreed). The following statements were used:

- I am confident when I support the children in their math development.
- It's easy to incorporate mathematics into everyday activities.
- I have sufficient knowledge of mathematics to be able to help the children learn mathematics.
- I have sufficient knowledge of mathematics teaching and learning to be able to help the children learn mathematics.
- I draw the children's attention to mathematics in many different activities.
- I have a mathematical language that helps me to draw the children's attention to mathematics in different activities.
- I can link activities in preschool to specific mathematical phenomena.
- It is easy to introduce mathematical phenomenon into different activities.
- I feel self-assured supporting the children in their learning of mathematics.

How the noticing ability received from the PD program influenced their EP was measured using a self-reported final reflection.

Data Analysis

The first data analysis was made on the preschool teachers reports, where they described mathematics in preschool activities. To explore the first research question, several analyses were conducted. All written reports by preschool teachers were reviewed to identify intellectual artefacts present within them. These artefacts were then categorized into three groups:

(a) proficiency-related artefacts (e.g., conceptual understanding and procedural fluency),

(b) arithmetic-related artefacts (e.g., counting, ordinality, and cardinality), and

(c) geometry-related artefacts (e.g., over, under, and behind).

Subsequently, the number of intellectual artefacts related to proficiency, arithmetic, and geometry were counted in each report and summarized. This process was repeated for both pre-project and post-project reports. Consequently, we obtained values representing the occurrence of intellectual artefacts in each report before and after the project. To analyze differences between the pre- and post-measures of intellectual artefacts related to proficiency, arithmetic, and geometry, paired sample t-tests were performed.

The second data analysis focused on preschool teacher's efficacy beliefs. The questionnaire was adapted to reflect preschool teacher's efficacy beliefs concerning supporting mathematics learning. We conducted an exploratory factor analysis to examine the relationships between variables and determine if our observed variables were correlated. This analysis revealed a reduced set of unobserved variables, referred to as factors. Naming these factors involves identifying the highest loadings, a common approach in research (Magnusson, 2003). Nine statements were used to measure preschool teachers' efficacy beliefs. Principal component analysis followed by varimax rotation on all statements revealed two factors at the pre-test (67.5% of variance explained; eigenvalues = 12.7; 3.4) and two factors at the post-test (69.2% of variance explained; eigenvalues = 14.4; 1.6). All statements

Table 1. Factors, statement, and factor loadings on pre- and post-test

Factor	Statement	Loading	
		Pre-test	Post-test
CMT	I have sufficient knowledge of mathematics to be able to help the children learn mathematics.	.822	.799
	I have sufficient knowledge of mathematics teaching and learning to be able to help the children learn mathematics.	.734	.735
	I have a mathematical language that helps me to draw the children's attention to mathematics in different activities.	.602	.621
	I can link activities in preschool to specific mathematical phenomena.	.595	.584
EP	I am confident when I support the children in their math development.	.902	.920
	It is easy to introduce mathematical phenomena into different activities.	.735	.754
	I draw the children's attention to mathematics in many different activities.	.721	.714
	It is easy to incorporate mathematics into everyday activities.	.666	.654
	I feel self-assured in supporting the children in their learning of mathematics.	.621	.644

Table 2. Mean usage of intellectual artefacts pre- and post-program

	Pre-program		Post-program	
	M	SD	M	SD
Intervention (n = 44)				
Proficiency	1.63	.13	4.54	.23
Arithmetic	3.86	.16	15.71	.27
Geometry	3.22	.22	12.45	.27
Control (n = 43)				
Proficiency	1.42	.14	1.39	.12
Arithmetic	3.62	.21	3.87	.19
Geometry	3.28	.18	3.43	.12

Note. M: Mean & SD: Standard deviation

Table 3. Self-efficacy supporting learning of mathematics pre- and post-program

	Pre-program		Post-program	
	M	SD	M	SD
Intervention (n = 44)				
CMT	4.31	.25	6.05	.23
EP	3.86	.19	6.45	.24
Control (n = 43)				
CMT	4.11	.27	4.19	.17
EP	4.21	.23	4.34	.22

Note. M: Mean & SD: Standard deviation

loaded at the same factor concerning pre- and post-test. The factors presented in **Table 1** were named

(a) content knowledge for mathematics teaching (CMT) and

(b) EP.

Means for each factor were calculated and pre- and post-test measures and repeated measures of analysis were used to detect development differences between groups.

The third research question was analyzed using a correlation test to investigate the relationship between preschool teachers' ability to notice mathematics in preschool activities and their self-efficacy in supporting learning in mathematics. The analysis was made on both pre- and post-scores.

In addition to analyzing quantitative data, we examined teachers' responses in their final reflections. Our focus was on identifying patterns in teachers' observations of their learning, EP, and their children's learning experiences. We aimed to uncover anecdotal statements that highlighted the connections between knowledge concerning intellectual artefacts, beliefs, and activities in preschool. To analyze the preschool teachers' self-reports, Braun and Clarke's (2006) thematic analysis was used. In the first phase of the analysis, the transcripts were coded to describe how the PD had affected their learning, EP, and their children's learning experiences. In the next step, these codes were then clustered into themes. The codes within each theme were then compared to find specific factors that were deemed to be essential to the theme. In the last analytical process, we searched for confirming and disconfirming statements concerning their learning, EP, and their children's learning experiences (cf. Erickson, 1986).

RESULTS

The results will be presented in two sections. The first section focuses on the development of preschool teachers' ability to use intellectual artefacts and their self-efficacy before and after our PD program. The second section focuses on qualitative trends observed in the reflections of preschool teachers who attended the PD program.

Development of Preschool Teachers' Capacity to Teach Mathematics

Table 2 and **Table 3** provide descriptive statistics for preschool teachers' ability to notice mathematics in preschool activities and their self-efficacy in supporting mathematics learning in preschool concerning CMT and EP.

Results show that preschool teachers involved in the PD program used significantly more intellectual artefacts related to proficiency $t(43) = 4.55$, $p < .001$), arithmetic $t(43) = 38.57$, $p < .001$), and geometry $t(43) = 33.48$, $p < .001$) when describing mathematics in preschool activities after participating in the program than before. In the control group, no significant differences were observed regarding pre- and post-tests about proficiency $t(42) = .90$, $p = .37$), arithmetic $t(42) = 1.59$, $p < .12$), and geometry $t(42) = 1.64$, $p < .09$). Thus, the PD program affected the preschool teachers' ability to notice aspects of mathematics in preschool activities, as significant differences on pre- and post-tests were identified. In the next step of the analysis, I examined to what extent the PD program affected the development of preschool teachers' self-efficacy concerning supporting mathematics learning in preschool.

To investigate whether there was any difference regarding the development of preschool teachers' CMT between the intervention group and control group, we used an ANOVA with total scores for CMT as the dependent measure (**Figure 2**).

Figure 2. Development of CMT (Source: Author)

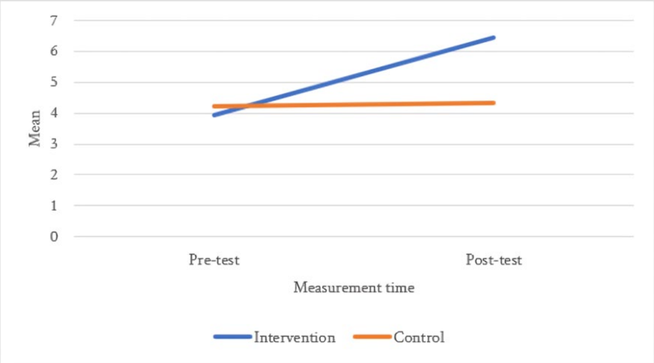

Figure 3. Development of pre-school teachers' EP (Source: Author)

Table 1. Correlation matrix

Variable	Proficiency	Arithmetic	Geometry	CMT	EP
Proficiency	1	.668**	.569**	.445**	.423*
Arithmetic		1	.354*	.543**	.543**
Geometry			1	.421*	.432*
CMT				1	.888**
EP					1

Note. $^{*}p < .05$; $^{**}p < .01$

This revealed a significant interaction effect between group and time $F(1, 85) = 93.41$, $p < .001$, $\eta^2 = 0.524$). There was also a main effect of time $F(1, 85) = 130.19$, $< .001$, $\eta^2 = 0.605$) and a main effect of group $F(1, 85) = 14.37$, $p < .001$, $\eta^2 = 0.143$).

A second ANOVA with total scores of EP as a dependent measure revealed a significant interaction effect between group and time $F(1, 85) = 104.11$, $p < .001$, $\eta^2 = 0.624$) (**Figure 3**). There was also a main effect of time $F(1, 85) = 99.19$ $p < .001$, $\eta^2 = 0.505$) and a main effect of group $F(1, 85) = 18.37$, $p < .001$, $\eta^2 = 0.213$).

In summary, our PD program influenced both the use of intellectual artefacts by preschool teachers in describing mathematics activities and their self-efficacy regarding CMT, as well as their implementation in practice.

Relation Between the Use of Intellectual Artefacts and Self-Efficacy

A correlation test conducted with pre-scores revealed no significant relationship between preschool teachers' use of intellectual artefacts to describe mathematics activities and their self-efficacy in supporting mathematics learning (**Table 4**). But correlation test conducted with postscores demonstrated a significant relationship between all measures of preschool teachers' use of intellectual artefacts to describe mathematics activities and their self-efficacy (CMT and EP). The strength between variables varies between small and strong relations.

Qualitative Trends in Teacher Reflection

Upon the conclusion of the program, preschool teachers were invited to contemplate the development of their knowledge and assess how the program influenced their own learning, EP, and their children's learning experiences.

Preschool teachers' knowledge and confidence

The analysis of the reports shows that the preschool teachers have developed their mathematical content knowledge as well as their pedagogical content knowledge which have made them more confident in supporting mathematics in preschool.

A recurring reflection from the teachers is that the program has affected their knowledge. The teachers believe that the program has led to "a better understanding of foundational mathematics they should support in preschool." "I understand more of how different parts of mathematics are related to each other."

When the teachers discuss their knowledge, they describe how it has affected the planning work, the work together with children but also their ability to reflect on the work of supporting the children in their mathematics learning. When the teachers discuss the knowledge, they emphasize that they can now see nuances in mathematics. They can differentiate between mathematics and mathematics, for example ordinality and cardinality. They also believe that this knowledge means that they as a collegium can be clearer in their planning work in terms of what we should focus attention on, but also how we should do it. "We are more focused in our planning, with respect to activities and mathematics." Another result of the newly acquired knowledge allows teachers to reflect on their activities in a more qualified way. "The theoretical tools I have gained help me reflect on the activities and the children's mathematics knowledge."

The reports also show that preschool teachers are more confident in supporting children's mathematics learning. The teachers say that they feel "more professional when they can discuss their activities in a more nuanced way." "My self-esteem has also been strengthened by the fact that I am more prepared for mathematics in the activities that we engage in." The increased awareness of what they do makes them feel more confident and professional in their professional role. "I am more confident now, when I am aware of what I'm doing, simply more professional."

Enactment in practice

If the activities together with the children are specifically studied, some qualitatively diverse ways emerge that the preschool teachers' work describes has been affected:

(a) more mathematics is highlighted in a specific activity,

(b) more activities where mathematics is highlighted, and

(c) better follow-up of children's mathematics in different activities.

Through discussions in the program, we have become aware of the mathematics that can occur in various activities in preschool. This has meant that we are more prepared for what mathematics can occur in different activities, which has led to us paying attention to the children to more mathematics in different activities. "I see more math in different activities." "There is a lot of mathematics in every activity that we do in preschool. Now I can see it and can make the children aware of it and follow up on their own thoughts that are linked to mathematics." Several teachers argue that they follow up mathematics in a more qualitative way as well as in a quantitative way. "I think I both follow up and make more mathematics visible, and I follow up mathematics in a more distinct way."

Mathematics has previously been given attention in various planned activities, such as when "we gather where we count students or when we set the table or when we line up in a line." "But now it's also in play activities." Engaging in play is an inherent and crucial element in the educational journey of a preschooler. Now, play activities have become a potential of play-based learning as a potent method to impart mathematical knowledge. "By incorporating games, collaborative activities, and interactive play, children can build a robust foundation in key mathematical concepts like counting, sorting, and spatial relationships." Some preschool teachers express surprise at how much mathematics there seems to be in preschool activities. "After all, there is mathematics all around us."

To sum up: The qualitative results strengthen the quantitative results. The preschool teachers testify that they have more knowledge about mathematics in preschool, which has made them feel more secure at the same time as they describe that they pay more attention to mathematics in their everyday activities.

DISCUSSION

In this study, we investigated the effects of a PD program focusing on preschool teachers' ability to notice mathematics in preschool activities. Our PD program was developed with the support of the Darling-Hammonds et al. (2017) review where they outlined central components that need to be considered in the PD program.

The group that attended the program developed their ability to notice mathematics significantly between the pre- and post-test, measure of the use of intellectual artefacts describing mathematics activities. One of the key tasks of preschool teachers is to establish a conducive setting that fosters children's mathematical cognition at their developmental level. This entails arranging a classroom with suitable resources and preschool teachers that are sensitive to integrating mathematics into everyday activities. To be sensitive to integrating mathematics teachers need to notice the mathematics that occurs in the activities (cf. Kaiser & König, 2019; Santagata et al., 2021; Scheiner, 2016). Teachers engage in a multifaceted process that includes noticing, interpreting information, and making decisions regarding aspects like what to focus on, how to interpret student actions, and how to facilitate further learning (Tatto et al., 2008). Santagata et al. (2021) contend that socio-cultural studies have broadened the exploration of this process by examining the role of artefacts that shape and guide teacher observation. In our PD program intellectual artefacts were implemented which gave the preschool teacher an extended ability to notice mathematics in everyday activities and mediate mathematics to the children. This can lead to children who do not pay attention to mathematics spontaneously being given additional opportunities to be involved in mathematical activities. Hannula and Lehtinen (2005) argue that this is important as support of early spontaneous attention to numbers and counting is related to later counting ability.

The results of the study also show that preschool teachers' self-efficacy concerning CMT and EP increases. In our PD program, the preschool teachers were given an opportunity to conduct and analyze their teaching while also having the opportunity to observe their colleagues. They had also the opportunity to discuss with colleagues and experts what mathematics occurred in different activities. Thereby they emphasized at least three of four factors that Bandura (1997) argues support the development of efficacy beliefs

(a) enactive mastery experience,

(b) vicarious experience and

(c) verbal persuasion.

Thereby our results strengthen Bandura's (1997) theory of what factors affect self-efficacy beliefs outcome. Preschool teachers' self-efficacy also correlates with the use of intellectual artefacts describing mathematics in everyday activities after the PD program. One interpretation is that an increased ability to notice mathematics in preschool activities enhances the preschool teachers' sense of competence (cf. Bandura, 1977).

Our results confirm that a PD program where Darling-Hammonds et al. (2017) components are contemplated, shows a positive effect targeting preschool teachers' PD regarding noticing ability. With support of earlier research, we argue that our results, increased ability to notice and self-efficacy, will have an impact on children's learning of mathematics, thou several researchers have shown that effective teaching in mathematics is influenced by both teachers' knowledge and their beliefs about teaching and learning (Ball et al., 2001; Berlin et al., 2021; Hill et al., 2005; Remillard, 2005; Richardson, 1996; Shulman, 1986; Thompson, 1992).

Our results are promising, though they come with certain limitations. One notable limitation is the sample size: despite having a small sample, we observed significant effects related to participation in our PD program. However, a larger sample would have provided a stronger foundation for more robust findings. Another limitation is the limited prior research on developing noticing abilities within a PD program. A more extensive body of literature in this area would have better informed our understanding of the research question. Therefore, this limitation also highlights an opportunity to address gaps in existing literature.

CONCLUSION

In conclusion, this work has clear implications for future research and practice. The PD program offered several ingredients that gave preschool teachers opportunities to develop their knowledge and beliefs. Internalization of intellectual artefacts, practice noticing mathematics in everyday activities and opportunities for common reflection are ingredients we interpret had an impact on the outcome. Thus, with support of our PD program the preschool teachers developed their ability noticing as well as their self-efficacy beliefs regarding CMT, as well as their implementation in practice. The preschool teachers report having gained greater knowledge about mathematics in the preschool setting, which has increased their confidence. They also describe being more attentive to incorporating mathematics into their daily activities.

More research can be done to investigate how programs focusing on teachers' noticing ability affect knowledge and beliefs in other subject domains, and how it serves in other contexts and grade levels. It would also be interesting to observe how mathematics is made visible in preschool activities when preschool teachers participate in PD programs.

To enhance preschool teachers' ability to teach mathematics to preschoolers, they should actively engage in PD. Staying abreast of current research in early childhood education and collaborating with colleagues fosters a dynamic and informed teaching approach. This commitment to ongoing learning ensures that teachers remain well-equipped to meet the evolving needs of preschoolers.

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